Pearson

Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE Further Mathematics
Statistics S4 (6686)

edexcel

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Summer 2017
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General Marking Guidance

- ☐ All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- ☐ Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- ☐ Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- ☐ There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- ☐ All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- ☐ Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- ☐ When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- ☐ Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $*$ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1. (*a*)	H_0: $\sigma_B^2 = \sigma_G^2$, H_1: $\sigma_B^2 \neq \sigma_G^2$,	B1
	$[s_B^2 =] \frac{1}{8}(4693.6 - 9 \times 22.8^2) = 1.88$	M1
	$[s_G^2 =] \frac{1}{5}(5236.12 - 6 \times 29.5^2) = 2.924$ awrt 2.92	A1
	$\frac{s_G^2}{s_B^2} = 1.555\ldots[0.643]$	M1 A1
	critical value $F_{5,8} = 3.69[0.271]$	B1
	not significant, variances are the same	A1 cso (7)
(*b*)	H_0: $\mu_G = \mu_B + 5$, H_1: $\mu_G > \mu_B + 5$	B1
	pooled estimate of variance $s_p{}^2 = \frac{8\times1.88 + 5\times2.924}{13} = 2.2815\ldots$ or $s_p = 1.51046\ldots$	M1
	test statistic $t = \pm\left(\frac{29.5 - 22.8 - 5}{s\sqrt{\frac{1}{9}+\frac{1}{6}}}\right) = \pm$ awrt 2.14 or $p = 0.0262$	M1 M1A1
	critical value $t_{13}(1\%) = \pm 2.650$ or $0.0262 > 0.01$	B1
	Insufficient evidence to support **Headteacher**'s claim or The **time** taken for girls is **not more than 5** seconds greater than for boys	A1 cso (7)
	Notes	Total 14
(a)	B1 both hypotheses. Must use σ or σ^2 and make clear which is H_0 and which is H_1. Do not allow in words M1 correct method for either s_B^2 or s_G^2 A1 Both s_B^2 and s_G^2 correct to 3sf allow sd's M1 allow use of s_B and s_G instead of s_B^2 or s_G^2 A1 awrt 1.56 or 0.643 B1 correct CV for their *F* or a correct comparison if use *p* A1 cso – All previous marks must be awarded. Variances are the same or var are not different	
(b)	B1 both hypotheses using μ. Do not allow $\geq$ sign instead of >. May use different letters eg A and B but they must be defined. M1 only allow use of s_B and s_G instead of s_B^2 or s_G^2 - May be seen in part(a) M1 use of correct formula with their s_p – condone missing 5 M1 use of correct formula with their s_p. (which must have been attempted) B1 correct CV but must match *t*-value or a correct comparison if use *p* A1 A correct statement with either the word Headteacher/Teacher/Head or time and not more than 5 oe do not allow contradicting statements.	

Question Number	Scheme	Marks
2.(a)	$H_0 : \lambda = 6,\ H_1 : \lambda > 6$ both	B1
	$P(X \geq 10) = 0.0839$	M1
	$P(X \geq 11) = 0.0426$	
	CR $X \geq 11$	A1
	P (Type I Error) = 0.0426	A1
		(4)
(b)	9 is not in the critical region therefore there is no evidence of an increase in the number of accidents per year **or** there is no evidence to support Jonty's claim	M1 A1ft
		(2)
(c)	λ=8	
	$P(X \leq 10 \mid \lambda = 8) = 0.8159$	M1A1
		(2)
	Notes	**Total 8**

(a) B1 both hypotheses, allow use of μ

M1 for seeing $[P(X \geq 10) =]0.0839$ or $[P(X \geq 11) =]0.0426$ or $[P(X \leq 9) =]0.9161$ or $[P(X \leq 10) =]0.9574$ oe allow a sideways slip of 1. ie 6.5/5.5

A1 for seeing $P(X \leq 10) = 0.9574$ or $P(X \geq 11) = 0.0426$ or CR $X \geq 11$

A1 0.0426

NB An answer of 0.0426 implies will get M1A1A1

(b) M1 must have 9/ value oe is not in CR allow 0.153 > 0.05

A1ft correct statement in context – need accidents or Jonty

(c) M1 $P(X \leq c-1 \mid \lambda = 8)$ with $c - 1$ being correct or using their c. Allow if a CR is stated in the form $X \leq c$ for $1 - P(X \leq c \mid \lambda = 8)$

A1 awrt 0.816

Question Number	Scheme	Marks
3(a)	H_0: $\mu = 135$ H_1: $\mu < 135$	B1
	$\bar{x} = 131$ $\quad s^2 = 10$	B1 B1
	$t = \dfrac{131-135}{\sqrt{10/5}} = -2.828\ldots$	M1A1
	critical value $t_4(10\%) = -1.533$	B1
	sufficient evidence that the **mean length** of wing is less than **135** mm.	A1 ft
		(7)
(b)	90% CI is given by	
	$\dfrac{4\times10}{9.488} < \sigma^2 < \dfrac{4\times10}{0.711}$	M1 B1B1
	(4.22, 56.3)	A1 (4)
	Notes	Total 11
(a)	B1 Both hypotheses	
	B1 131	
	B1 10 or awrt 3.16	
	M1 Allow $\pm \dfrac{\text{"}their\ 131\text{"}-135}{\sqrt{\text{"}their\ 10\text{"}/5}}$	
	A1 awrt -2.83 or $-2\sqrt{2}$	
	B1 ± 1.533 sign must match t-value or be $\pm$	
	A1ft ft t-value if awarded 1st and 4th B marks. The words 'mean length' and '135' must be included in the context	
(b)	M1 $\dfrac{4\times\text{"their 10"}}{\chi^2 value}$	
	B1 awrt 9.49	
	B1 awrt 0.711	
	A1 awrt 4.22/4.21 and awrt 56.3	

Question Number	Scheme	Marks
4(a)(i)	The data is collected in pairs or samples not independent	B1
(ii)	The differences are normally distributed	B1
		(2)
(b)	d: 6 2 -3 3 4 4 -2 3	M1
	($\Sigma d = 17$, $\Sigma d^2 = 103$) $\bar{d} = \pm 2.125$, $s_d = 3.09\ldots\ldots$ (Var = 9.55…)	M1 M1
	H_0: $\mu_d = 1$, H_1: $\mu_d > 1$ (H_0: $\mu_d = -1$ H_1: $\mu_d < -1$ if differences are - 6, -2, 3 etc)	B1
	$t = \pm\left(\dfrac{2.125-1}{3.09/\sqrt{8}}\right) = \pm 1.02947\ldots\ldots$	M1A1
	Critical value $t_7(5\%) = \pm$ 1.895 (1 tail)	B1
	Not significant. Insufficient evidence to support that the **score** in the **final** round is **more than 1 below** the score in the **first** round or insufficient evidence to support the **coach**'s belief.	A1ft
		(8)
	SC for two sample test they may get M0M0 M0B1M0A0B1A0 H_0: $\mu_{first} = \mu_{final} + 1$, H_1: $\mu_{first} > \mu_{final} + 1$, $\pm$ 1.761	
(c)	The idea that "the coach's belief is rejected when it is in fact true"	B1 B1
		(2)

	Notes	Total 12
(a)(i)	B1 Allow because the same person has been used. Do not allow 2 data sets.	
(ii)	B1 for a comment that mentions "differences" and "normal" distribution	
(b)	M1 for attempting the ds, at least 2 correct implied by the figures ($\Sigma d = 17$, $\Sigma d^2 = 103$, $\bar{d} = \pm 2.125$, $s_d = 3.09\ldots\ldots$) M1 for attempting $\bar{d}$ M1 for s_d or $s_d{}^2$ B1 for both hypotheses correct in terms of μ or μ_d (allow a defined symbol) Must match their differences M1 for attempting the correct test statistic $\dfrac{\bar{d}-1}{s_d/\sqrt{8}}$ A1 awrt 1.03 B1 awrt 1.895 sign must match their t-value A1ft ft t-value if awarded both B marks. A correct comment in context – bold words needed.	
(c)	B1 for H_1 is rejected when it is in fact true B2 Correct contextual statement.	

Question Number	Scheme	Marks
5	$\bar{x} = \dfrac{492+507}{2}$	M1
	$= 499.5$	A1cao
	$2.093\dfrac{s}{\sqrt{20}} = 7.5$	M1,B1
	$s = 16.02533...$ ($s^2 = 256.81..6$)	A1
	$s_p^2 = \dfrac{19\times 16.025..^2 + 9\times 280}{28} = 264.26\ldots$	M1A1ft
	$t_{28(0.05)} = 1.701$	B1
	90% CI = $(499.5 - 480) \pm 1.701 \times \sqrt{264.26} \times\sqrt{\dfrac{1}{20}+\dfrac{1}{10}}$	M1A1ft
	$= (8.8, 30.2)$	A1cao (11)
	Notes	
	M1 $\bar{x} = \dfrac{492+507}{2}$ A1 499.5 cao M1 $t-\text{value}\dfrac{s}{\sqrt{20}} = 7.5$ B1 2.093 A1 awrt 16.0 for s or 257 for s^2 M1 $\dfrac{"n_1-1"\times\left(s \text{ or } s^2\right)+"n_2-1"\times\left(s \text{ or } s^2\right)}{n_1+n_2-2}$ finding s_p^2 A1 ft their s^2 B1 awrt 1.701 M1 $(\bar{x}-480) \pm t\text{-value} \times \sqrt{s_p{}^2} \times\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}$ A1 ft their s_p^2 and $\bar{x}$ A1cao awrt 8.8 and awrt 30.2	

Question Number	Scheme	Marks
6(a)	$\mathrm{E}\left(\frac{aX_1+bX_2}{n}\right)=\frac{anp+bnp}{n}=ap+bp=(a+b)p$	**M1**
	$a+b=1$ *	**A1* cso** **(2)**
(b)	$\mathrm{Var}\left(\frac{aX_1+bX_2}{n}\right)=\frac{1}{n^2}\left(a^2np(1-p)+b^2np(1-p)\right)$	**M1 A1**
	$=\frac{p(1-p)\left(a^2+b^2\right)}{n}$	
	$=\frac{p(1-p)\left(a^2+(1-a)^2\right)}{n}$	**M1d**
	$=\frac{\left(2a^2-2a+1\right)p(1-p)}{n}$ *	**A1* cso** **(4)**
(c)	Min value when $\frac{(4a-2)p(1-p)}{n}=0$	**M1A1**
	$\Rightarrow 4a-2=0$	
	$a=\frac{1}{2},\ \ b=\frac{1}{2}$	**A1A1ft**
	$\frac{\mathrm{d}^2\mathrm{Var}(\hat{p})}{\mathrm{d}a^2}=\frac{4p(1-p)}{n}>0$ or ∵ quadratic with positive x^2 ∴ minimum point or sketch	**B1** **(5)**
(d)(i)	$\mathrm{E}\left(\frac{aX_1+bX_2}{n}\right)^2=\mathrm{E}\left(\frac{a^2X_1{}^2+b^2X_2{}^2+2abX_1X_2}{n^2}\right)$	**M1**
	$=\frac{1}{n^2}\left(a^2np(1-p)+a^2n^2p^2+b^2np(1-p)+b^2n^2p^2+2abn^2p^2\right)$	**M1d**
	$=\frac{(a^2+b^2)np(1-p)+(a+b)^2n^2p^2}{n^2}$	
	$=\frac{(a^2+b^2)p(1-p)}{n}+p^2(a+b)^2$	
	$=\frac{(a^2+b^2)p(1-p)}{n}+p^2$; $>p^2$ since $\frac{(a^2+b^2)p(1-p)}{n}>0$ oe ∴ biased	**A1;A1**
(ii)	As $n\to\infty$ $\mathrm{E}\left(\hat{p}^2\right)\to p^2$ Therefore bias $\to 0$	**B1** **(5)**
(e)	$\mathrm{E}\left(X_1(X_1-1)\right)=\mathrm{E}\left(X_1{}^2\right)-\mathrm{E}\left(X_1\right)$	
	$=np(1-p)+n^2p^2-np$	**M1**
	$=np-np^2+n^2p^2-np$	
	$=np^2(n-1)$	**A1**
	Unbiased estimator = $\frac{X_1(X_1-1)}{n(n-1)}$	**A1** **(3)** **Total 19**

	Notes	
(a)	M1 Using $\frac{a\mathrm{E}(X_1) + b\mathrm{E}(X_2)}{n}$ and subst $\mathrm{E}(X_1) = np$ and $\mathrm{E}(X_2) = np$	
	Acso* Answer given . Need $p(a + b) = p$ and statement $a+b=1$ and no errors	
(b)	M1 Using $\frac{a^2\mathrm{Var}(X_1) + b^2\mathrm{Var}(X_2)}{n^2}$ and subst $\mathrm{Var}(X_1) = np(1-p)$ – may be implied by $\frac{1}{n^2}\left(a^2np(1-p)+b^2np(1-p)\right)$ A1 correct answer in any form M1d dep on 1st M1 Subst $b = 1 - a$ A1cso* method must be shown and no errors.	
(c)	M1 $\frac{\mathrm{d}}{\mathrm{da}}(\mathrm{Var})$ (must differentiate with respect a) or attempt to complete the square A1 correct diff = 0 or $2\left(a-\frac{1}{2}\right)^2+\frac{1}{2}$ A1 $a = 0.5$ A1 ft for $b = 1 - a$ B1 for a reason why minimum	
(d)(i)	M1 multiplying out and using $\mathrm{E}(aX) = a\,\mathrm{E}(X)$ [may use their values of a and b] M1d dependent on previous M being awarded Using $\mathrm{E}(X^2) = \mathrm{Var}(X) + [\mathrm{E}(X)]^2$ A1 $\frac{(2a^2-2a+1)p(1-p)}{n}+p^2$ or $\frac{(a^2+b^2)p(1-p)}{n}+p^2$ must be of the form p^2 + a single term A1 for a reason why it is not equal p^2 plus statement to say biased.	
(ii)	B1 Follow on from their expression $p^2 + \ldots$ with a and b.	
(e)	M1 multiplying out correctly and subst np for $\mathrm{E}(X)$ or using $\mathrm{E}(\hat{p})^2 = \mathrm{Var}(\hat{p}) + \left[\mathrm{E}(\hat{p})\right]^2$ Allow $= \frac{(2a^2-2a+1)p(1-p)}{n}+p^2$ A1 $np^2(n-1)$ A1 $\frac{X_1(X_1-1)}{n(n-1)}$	
	NB $\frac{X_1(X_1-1)}{n(n-1)}$ gains all 3 marks.	

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